DISSIPATIVITY AND GLOBAL ANALYSIS OF LIMIT CYCLES IN NETWORKS OF OSCILLATORS

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Abstract: This paper is concerned with the global analysis of synchrone oscillations in special networks of oscillators. In previous work, we defined a class of high-dimensional, parameter-dependent nonlinear systems exhibiting almost globally asymptotically stable limit cycle oscillations. In this paper, we show how (incremental) dissipativity may be used to extend the global analysis of limit cycle oscillations to networks of coupled identical systems.

Keywords: Stability analysis, limit cycles, networks of oscillators.

1. INTRODUCTION

Nonlinear oscillations are ubiquitous in nature. A complex system made up of coupled oscillatory systems can be considered as a large-scale network of coupled oscillators. In this paper, we show how global synchrone oscillations may be obtained by the interconnection of identical systems exhibiting globally attractive limit cycles when isolated.

We first recall previous results presented in [10] and [8]. These results characterize classes of parameter-dependent high-dimensional systems exhibiting almost globally attractive limit cycle oscillations. These classes of systems constitute generalizations of the well known Van der Pol and FitzHugh-Nagumo oscillators.

In the present paper, we show how the global stability analysis for one oscillator of type (3) extends to the global stability analysis of a synchrone oscillation in a network of N identical oscillators linearly coupled through their outputs.

Our approach consists in showing that, for strong enough coupling, all solutions of these particular networks exponentially converge to the invariant subspace

$$\{X \in \mathbb{R}^{nN} : x_1 = \cdots = x_N\} \tag{1}$$

where $X = (x_1, \ldots, x_N)^T$ denotes the state vector of the network.

In our approach, the synchronization property (1) derives from an incremental dissipativity property of the network. This approach is closely related to the contraction approach of [9] and [5]. The incremental dissipativity property together with an observability assumption may be seen as an incremental stability property of the particular network [1].

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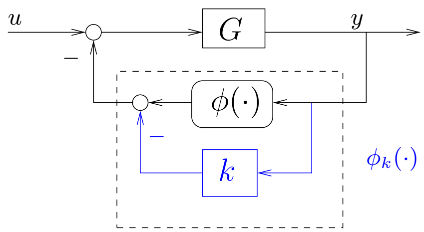

Fig. 1. Block diagram representing the class of SISO nonlinear systems.

Because the dynamics of the network decouple in the invariant subspace (1), the result implies that all oscillators synchronize asymptotically and that all bounded solutions converge to the ω-limit sets of the decoupled system.

Using the results of [2], we prove that all solutions of the coupled system are bounded.

Combining boundedness of the solutions and synchronization, almost global asymptotic stability of the limit cycle for an isolated system is then generalized to the situation when such identical systems are coupled into a network.

Simulations of networks of identical oscillators coupled according to different topologies are provided to illustrate the results of the paper.

2. THE DISSIPATIVE OSCILLATOR

In [8], we consider the feedback system shown in Figure 1 where the SISO passive system G is described by a linear and detectable state space model (A, B, C) whereas $\phi_k(\cdot)$ is the static nonlinearity

$$\phi_k(y) = -ky + \phi(y) \tag{2}$$

where $\phi(\cdot)$ is a smooth sector nonlinearity in the sector $(0, \infty)$, which satisfies $\phi'(0) = \phi''(0) = 0$, $\phi'''(0) > 0$ and $\lim_{|y|\to\infty} \frac{\phi(y)}{y} = \infty$ ("stiffening nonlinearity").

The resulting feedback dynamical equations write

$$\begin{cases} \dot{x} = Ax - B\phi_k(y) + Bu \\ y = Cx \end{cases} \tag{3}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ represents respectively the state, the input and the output of the feedback system.

We note G_k the (positive) feedback interconnection of G with the feedback gain k. The feedback system is equally described as the feedback interconnection of G_k and the (strictly passive) nonlinearity $\phi(\cdot)$.

We define a *dissipative oscillator* as a system that admits the feedback representation in Figure 1 and which satisfies two conditions

(1) the feedback system satisfies the dissipation inequality $\dot{S} \leq (k - k^*_{passive})y^2 - y\phi(y) + uy$ where $S(x)$ represents the storage function associated to the feedback system and $k^*_{passive}$ is the critical value of k above which the system G_k looses passivity.
(2) when unforced ($u = 0$), the feedback system possesses a global limit cycle, i.e. a stable limit cycle which attracts all solutions except those belonging to the stable manifold of the origin.

The first condition necessarily holds since we assume that the forward block G is passive. Our previous papers [10,8] provide sufficient conditions for the second condition to be satisfied as well. In particular, denoting k^* the bifurcation value at which G_k looses stability, we showed that absolute stability of (3) at $k = k^*$ implies (generically) one of the two conditions :

- a supercritical Hopf bifurcation at $k = k^*$ and a global limit cycle for $k \gtrsim k^*$ [2]
- a supercritical pitchfork bifurcation at $k = k^*$ and a global bistability for $k \gtrsim k^*$

The Hopf scenario provides a dissipative oscillator in the vicinity of the bifurcation, i.e. for $k \gtrsim k^*$. The pitchfork scenario provides a bistable system in the vicinity of the bifurcation. This bistable system is turned into a relaxation oscillation by first-order dynamic extension. The resulting system is a dissipative oscillator as well.

A sufficient condition for absolute stability at $k = k^*$ is that the system G_k looses passivity and stability for the same value of the parameter k, i.e. when $k^* = k^*_{passive}$. Multipliers can be used to extend the result to the more general situation when $k^* > k^*_{passive}$ (see [8]).

In [11], we restrict ourselves to a piecewise linear version of system (3) and adapt numerical tools recently proposed in the literature [3] to prove the global asymptotic stability of the limit cycle for a large range of parameter values above k^*.

In the present paper, we show how the stability analysis for one dissipative oscillator extends to the stability analysis of a synchrone oscillation in a network of N identical dissipative oscillators linearly coupled through their outputs.

3. INTERCONNECTIONS

In this section, we define a particular class of interconnections and identify the main property required for the incremental dissipativity property of Section 4. In Section 4, we will prove that global synchronization of the interconnected dissipative

[2] $k \gtrsim k^*$ means k "slightly" greater than k^*, i.e. $k \in (k^*, k^* + \epsilon)$ where $\epsilon > 0$ is small

oscillators is a consequence of the incremental dissipativity property of the network.

We consider a network of N identical oscillators, linearly coupled through their outputs. Let $\Gamma \in \mathbb{R}^{N\times N}$ be the interconnection matrix. We assume that $\mathbf{1}$ (the vector $(1,\dots,1)^T \in \mathbb{R}^N$) belongs to the kernel of Γ. This is equivalent to the assumption that all rows of Γ sum to zero. Moreover, we assume that the rank of Γ is equal to $N-1$. This is equivalent to the assumption that the network is connected.

Note that our assumptions do not require that Γ is symmetric.

The assumptions on Γ imply

$$\bar{R}\Gamma = \begin{pmatrix} 0 & \mathbf{0}^T \\ \mathbf{0} & \tilde{\Gamma} \end{pmatrix} \bar{R} \tag{4}$$

where $\bar{R} = (I_N - (\mathbf{1}, 0_{N\times N-1}))$ is a projection matrix.

The class of interconnection matrices Γ is further assumed to be such that $\tilde{\Gamma}$ is positive definite (i.e. $x^T\tilde{\Gamma}_s x = x^T\frac{1}{2}(\tilde{\Gamma}+\tilde{\Gamma}^T)x > 0, \forall x \in \mathbb{R}^N\backslash\{0\}$). In the rest of the paper, we denote by $\lambda_{min}(\tilde{\Gamma}_s)$ the smallest eigenvalue of the symmetric part of $\tilde{\Gamma}$.

This is easily seen by noting that

$$\begin{aligned}
\bar{R}\Gamma &= \begin{pmatrix} 0 & \mathbf{0}^T \\ -\mathbf{1} & I_{N-1} \end{pmatrix} \Gamma \\
&= \begin{pmatrix} 0 & \mathbf{0}^T \\ \mathbf{0} & I_{N-1} \end{pmatrix} \begin{pmatrix} -1 & \mathbf{0}^T \\ -\mathbf{1} & I_{N-1} \end{pmatrix} \Gamma \\
&= \begin{pmatrix} 0 & \mathbf{0}^T \\ \mathbf{0} & I_{N-1} \end{pmatrix} \begin{pmatrix} 0 & * \\ \mathbf{0} & \tilde{\Gamma} \end{pmatrix} \begin{pmatrix} -1 & \mathbf{0}^T \\ -\mathbf{1} & I_{N-1} \end{pmatrix} \\
&= \begin{pmatrix} 0 & \mathbf{0}^T \\ \mathbf{0} & \tilde{\Gamma} \end{pmatrix} \bar{R}
\end{aligned}$$

The third equality comes from the properties of Γ. More precisely, we have

$$\begin{pmatrix} -1 & \mathbf{0}^T \\ -\mathbf{1} & I_{N-1} \end{pmatrix}^{-1} \Gamma \begin{pmatrix} -1 & \mathbf{0}^T \\ -\mathbf{1} & I_{N-1} \end{pmatrix} = \begin{pmatrix} 0 & * \\ \mathbf{0} & \tilde{\Gamma} \end{pmatrix}$$

since, by assumption, $\mathbf{1} \in \mathbb{R}^N$ belongs to the kernel of Γ. The third equality is then deduced from $\begin{pmatrix} -1 & \mathbf{0}^T \\ -\mathbf{1} & I_{N-1} \end{pmatrix}^{-1} = \begin{pmatrix} -1 & \mathbf{0}^T \\ -\mathbf{1} & I_{N-1} \end{pmatrix}$.

Property (4) constitutes the main characteristic of the class of interconnections we consider.

4. INCREMENTAL DISSIPATIVITY AND SYNCHRONIZATION

In this section, we prove global synchronization of oscillators of type (3) interconnected according to a topology satisfying (4). In our approach, synchronization results from an incremental dissipativity property of the network.

Consider a network of N identical dissipative oscillators. The dynamics for oscillator $i = 1,\dots,N$ write

$$\begin{cases} \dot{x}_i = Ax_i - B\phi_k(y_i) + Bu_i \\ y_i = Cx_i \end{cases} \tag{5}$$

where u_i represents the external input to oscillator i.

Each system i is characterized by the dissipation inequality

$$\dot{S}_i \leq (k - k^*_{passive})y_i^2 + u_i y_i \tag{6}$$

where $S_i = \frac{1}{2}(x^T P x)$, with $P = P^T > 0$, $PA + A^T P \leq 0$ and $PB = C^T$. S_i is the storage function associated with system i.

The dynamics of the network are easily represented with the help of the Kronecker product [4].

$$\begin{cases} \dot{X} = (I_N \otimes A)X - (I_N \otimes B)\,\Phi_k(Y) + (I_N \otimes B)\,U \\ Y = (I_N \otimes C)\,X \end{cases} \tag{7}$$

where $X = (x_1,\dots,x_N)^T$, $Y = (y_1,\dots,y_N)^T$, $\Phi_k(Y) = (\phi_k(y_1),\dots,\phi_k(y_N))^T$ and I_N represents the N by N identity matrix.

Assuming linear output coupling between the oscillators of the network, the MIMO external input of system (7) is

$$U = -\Gamma Y \tag{8}$$

where $\Gamma \in \mathbb{R}^{N\times N}$ represents the interconnection matrix defining the topology of the network.

The main result of this section is the following Theorem.

Theorem 1. Consider the MIMO system (7)-(8) representing a network of N identical oscillators of type (5) where (A, C) is observable and $\phi(\cdot)$ is monotone. Assume that each unforced oscillator ($u_i \equiv 0$) possesses a globally asymptotically stable limit cycle in $\mathbb{R}^n\backslash E_s(0)$ where $E_s(0)$ denotes the stable manifold of the origin. If the interconnection matrix Γ satisfies (4) with $\tilde{\Gamma} > 0$ then, for $\lambda_{min}(\tilde{\Gamma}_s) > k - k^*_{passive}$ (strong coupling), the network has a limit cycle which attracts all solutions except those belonging to the stable manifold of the origin.

□

PROOF. Consider the difference system

$$\begin{cases} \Delta\dot{X} = (I_N \otimes A)\Delta X - (I_N \otimes B)\,\Delta\Phi_k(Y) + (I_N \otimes B)\,\Delta U \\ \Delta Y = (I_N \otimes C)\,\Delta X \end{cases} \tag{9}$$

where $\Delta X = (\bar{R} \otimes I_n)X = (0, x_2 - x_1, \dots, x_N - x_1)^T$, $\Delta\Phi_k(Y) = \bar{R}\Phi_k(Y) = (0, \phi_k(y_2) - \phi_k(y_1), \dots, \phi_k(y_N) - \phi_k(y_1))^T$ and $\Delta U = \bar{R}U = -\bar{R}\Gamma Y$.

Property (4) implies

$$-\Delta Y^T \Delta U = \Delta Y^T \bar{R}\Gamma Y = \bar{Y}^T \tilde{\Gamma} \bar{Y} > 0,\ \forall \bar{Y} \neq 0 \tag{10}$$

where $\bar{Y} = (y_2 - y_1, \cdots, y_N - y_1)^T$.

Consider the storage function

$$S_\Delta = \frac{1}{2}\left(\Delta X^T \left(I_N \otimes P\right) \Delta X\right) \tag{11}$$

Using the properties of the Kronecker product, we obtain, successively, for its time derivative along the solutions of (9)

$$\begin{aligned}\dot{S}_\Delta &= \frac{1}{2}\left(\Delta X^T \left(\left(I_N \otimes P\right)\left(I_N \otimes A\right)\right.\right.\\ &\quad + \left.\left(I_N \otimes A^T\right)\left(I_N \otimes P\right)\right) \Delta X\Big)\\ &\quad -\Delta X^T \left(I_N \otimes P\right)\left(I_N \otimes B\right) \Delta\Phi_k(Y)\\ &\quad +\Delta X^T \left(I_N \otimes P\right)\left(I_N \otimes B\right) \Delta U\\ &\quad -k^*_{passive} \Delta Y^T \Delta Y\\ &= \frac{1}{2}\Delta X^T \left(I_N \otimes \left(PA + A^T P\right)\right) \Delta X\\ &\quad -\Delta X^T \left(I_N \otimes C^T\right) \Delta\Phi_k(Y)\\ &\quad +\Delta X^T \left(I_N \otimes C^T\right) \Delta U - k^*_{passive} \Delta Y^T \Delta Y\\ &\leq (k - k^*_{passive}) \Delta Y^T \Delta Y + \Delta Y^T \Delta U - \Delta Y^T \Delta\Phi(Y)\\ &\leq \bar{k} \Delta Y^T \Delta Y + \Delta Y^T \Delta U \end{aligned} \tag{12}$$

where $\bar{k} = k - k^*_{passive}$.

The second equality and the first inequality are simply a consequence of the passivity of each oscillator in the network. The second inequality comes from the monotone increasing property of $\phi(\cdot)$.

Inequality (12) expresses that the network satisfies a dissipativity inequality in terms of the Δ-variables, a property that we call *incremental dissipativity*.

Using (10) and (12) we deduce,

$$\begin{aligned}\dot{S}_\Delta &\leq \bar{k} \Delta Y^T \Delta Y + \Delta Y^T \Delta U\\ &= \left(\bar{k} - \lambda_{min}(\tilde{\Gamma}_s)\right) \Delta Y^T \Delta Y\end{aligned} \tag{13}$$

where $\lambda_{min}(\tilde{\Gamma}_s)$ is the smallest eigenvalue of the symmetric part of $\tilde{\Gamma}$.

From the strong coupling assumption,

$$\gamma = \lambda_{min}(\tilde{\Gamma}_s) - \bar{k} > 0 \tag{14}$$

Integrating (13) over $[t_0, t_0 + \delta]$, we obtain

$$\begin{aligned}\int_{t_0}^{t_0+\delta} \dot{S}_\Delta d\tau &\leq -\gamma \int_{t_0}^{t_0+\delta} |\Delta Y|^2 d\tau\\ &\leq -\alpha\gamma |\Delta X(t_0)|^2, \;\; \alpha > 0\end{aligned} \tag{15}$$

for all $\Delta X(t_0) \in \mathbb{R}^{nN}$, $t_0 \geq 0$. The last inequality comes from the observability of (A, C). GES of $\Delta X(t)$ is then deduced from classical exponential stability theorems (see, for example, [6, Theorem 1.5.2]).

GES of the solution $\Delta X = 0$ for the difference system (9) implies that all solutions of the network (7) exponentially converge to the invariant subspace

$$\{X \in \mathbb{R}^{nN} : x_1 = \cdots = x_N\} \tag{16}$$

where the dynamics are decoupled. Because the dynamics of the network decouple in the invariant subspace (16), GES of the solution $\Delta X = 0$ for the difference system (9) implies that all bounded solutions converge to the ω-limit sets of the decoupled system and that all oscillators synchronize asymptotically.

Finally, using the results of [2], we prove that all solutions of the coupled system are bounded. In fact, each oscillator (5) is input-to-state stable (ISS) with respect to u_i (see [2, Theorem 1]) and $|u_i| \leq K|\Delta Y|$. Since (9) is GES, $|\Delta Y|$ is bounded by a decreasing exponential function and we conclude from [2, Theorem 2] that for oscillator i the state $x_i(t) \in \mathbb{R}^n$ is bounded for all $i = 1, \ldots, N$.

Combining GES of the difference system (9) and boundedness of the solutions, we conclude that, for strong coupling, all solutions of the network (7) converge to the ω-limit sets of the uncoupled dynamics, i.e. all solutions except those belonging to the stable manifold of the origin of the network converge towards a unique limit cycle.

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Remark 2. The result still holds if the observability assumption on the pair (A, C) is relaxed to a detectability assumption.

Remark 3. The GES result of $\Delta X = 0$ may be viewed as an incremental input-to-state stability (δ-ISS) property of the network with S being the corresponding δ-ISS Lyapunov function [1].

Remark 4. Our approach is also strongly linked with Slotine's contraction theory to prove synchronization [9] and to Pogromsky's synchronization results [5]. This may easily be noticed from the normal form of passive systems.

The normal form for oscillator i of the network is [7]

$$\begin{aligned}\begin{pmatrix}\dot{z}_i\\ \dot{y}_i\end{pmatrix} &= \begin{pmatrix}Q & e\\ f^T & g\end{pmatrix}\begin{pmatrix}z_i\\ y_i\end{pmatrix} + \begin{pmatrix}0\\ CB\end{pmatrix}(ky_i - \phi(y_i))\\ &\quad - \sum_{j=1}^{N} \gamma_{ij} \begin{pmatrix}0 & 0\\ 0 & CB\end{pmatrix}\left(\begin{pmatrix}z_j\\ y_j\end{pmatrix} - \begin{pmatrix}z_i\\ y_i\end{pmatrix}\right)\end{aligned} \tag{17}$$

where CB is positive definite from the passivity assumption. Assume that $\gamma_{ij} \leq 0$ for $i \neq j$, then the couplings $-\gamma_{ij}\begin{pmatrix}0 & 0\\ 0 & CB\end{pmatrix}$ are positive semidefinite. The symmetric part of the jacobian of the uncoupled dynamics, divided according the coupling structure, is given by

$$J_{is} = \begin{pmatrix} Q_s & \frac{1}{2}(e+f) \\ \frac{1}{2}(e+f)^T & g + CBk - CB\frac{d\phi(y_i)}{dy_i} \end{pmatrix} \quad (18)$$

It is then easily seen that the sufficient conditions given by Slotine [9, Remark 3 of Theorem 2] are satisfied, i.e.

(1) Q_s is contracting since it is Hurwitz from the passivity and detectability assumptions
(2) $\lambda_{max}(g + CBk - CB\frac{d\phi(y_i)}{dy_i}) < g + CBk < \infty$ from the monotone increasing assumption of
(3) $\sigma_{max}\left(\frac{1}{2}(e+f)\right) = \left|\frac{e+f}{2}\right|^2 < \infty$

Note that the results in [9] or [5] require generically that $\gamma_{ij} = \gamma_{ji} \leq 0$ for $i \neq j$, an assumption that we do not make. Exploiting the special structure of dissipative oscillators, we additionally show that the limit cycle stability analysis carried for an isolated oscillator extends to the network.

5. EXAMPLES

In [8], we consider a nontrivial instance of dissipative oscillator of order 3 possessing a globally asymptotically stable limit cycle in $\mathbb{R}^3 \backslash E_s(0)$ for $k \gtrsim k^*$. The corresponding transfer function is $G(s) = \frac{s(\tau s + \omega_n^2)}{s^3 + 2\zeta\omega_n s^2 + (\tau + \omega_n^2)s + \omega_n^2}$ with $2\zeta\tau \geq \omega_n > 0$. The critical values k^* and $k^*_{passive}$ are given in (19) and (20).

As an illustration of Theorem 1, we consider a S_N-symmetry network of such dissipative oscillators. For S_N-symmetry coupling, we have

$$\Gamma = \begin{bmatrix} (N-1)K & -K & \cdots & -K \\ -K & (N-1)K & \cdots & -K \\ \vdots & \vdots & \ddots & \vdots \\ -K & -K & \cdots & (N-1)K \end{bmatrix} \quad (21)$$

where K is the coupling strength characterizing the S_N symmetry network.

In this particular case, it is easy to show that $\tilde{\Gamma} = diag(NK, \ldots, NK)$ and thus $\lambda_{min}(\tilde{\Gamma}_s) = NK$ with an algebraic multiplicity equal to $N-1$. From condition (14), synchronization is ensured if $K > \frac{k - k^*_{passive}}{N}$.

By Theorem 1, we conclude that for $K > \frac{k - k^*_{passive}}{N}$, all solutions, except those belonging to the stable manifold, converge towards the ω-limit set of the uncoupled system which is a globally attractive limit cycle for $k \gtrsim k^*$.

Simulation results for a network of 5 coupled oscillators are presented in Figure 2. For the

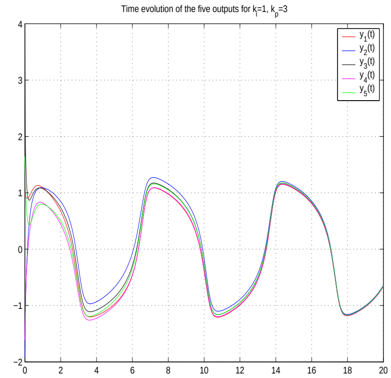

Fig. 2. Time evolution of the outputs in a network of 5 oscillators coupled through S_5 symmetry.

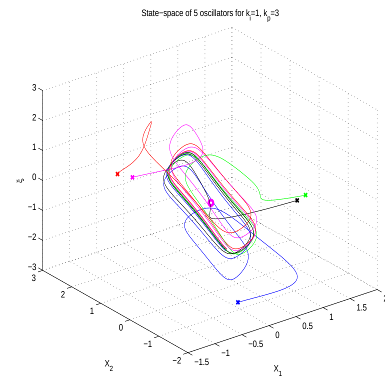

Fig. 3. Superposition of the state spaces of the 5 oscillators coupled through S_5 symmetry.

simulation, we chosed the following values of the parameters: $\tau = 2$, $\zeta = 2.5$ and $\omega_n = 1$. This leads to a critical bifurcation value $k^* = 2$ while the loss of passivity occurs at $k^*_{passive} = 1$. The nonlinearity $\phi(\cdot)$ we used in these simulations was $\phi(\cdot) = (\cdot)^3$. The initial conditions are chosen at random. The results concerning the existence of an almost globally asymptotically stable limit cycle in [8] hold only for $k \gtrsim k^*$. Nevertheless, we expect these results to hold valid for a (large) range of the bifurcation parameter k. To illustrate this we selected $k = 3$. For global synchronization, the common coupling strength K has to be strong enough (i.e., $K > \frac{3-1}{5} = 0.4$). For this simulation, the value of K was equal to 1.

On Figure 3, we clearly see that the oscillators synchronize around a common limit cycle oscillation. This limit cycle is identical to the one obtained for an isolated oscillator.

To illustrate the results for a non-symmetric coupling matrix Γ we consider a network of 3 identical dissipative oscillators. The interconnection matrix is

$$\Gamma = K \begin{bmatrix} 4 & 1 & -5 \\ 2 & 2 & -4 \\ 1 & 2 & -3 \end{bmatrix} \quad (22)$$

where $K > 0$.

$$k^* = \frac{\tau(\tau+\omega_n^2)+2\zeta\omega_n^3 - \sqrt{\tau^4+2\omega_n^2\tau^3+\omega_n^3(\omega_n-4\zeta)\tau^2+4\tau\omega_n^4(1-\zeta\omega_n)+4\zeta^2\omega_n^6}}{2\omega_n^2\tau} \quad (19)$$

$$k^*_{passive} = \min(1, \frac{2\zeta\omega_n\tau-\omega_n^2}{\tau^2}) \quad (20)$$

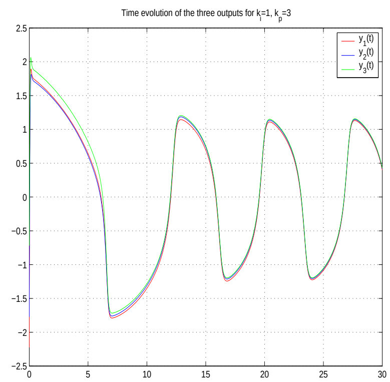

Fig. 4. Time evolution of the outputs in a network of 3 oscillators coupled according to (22).

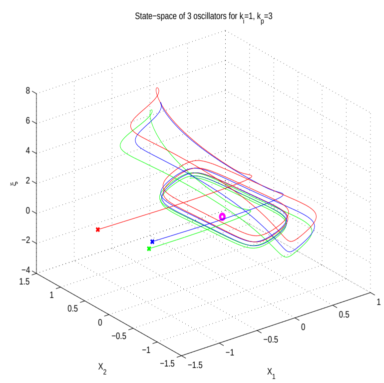

Fig. 5. Superposition of the state spaces of the 3 oscillators coupled according to (22).

In this case, $\tilde{\Gamma} = K\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$ and $\lambda_{min}(\tilde{\Gamma}_s) = \frac{3-\sqrt{5}}{2}K = 0.38K$

Chosing $K > \frac{k-k^*_{passive}}{0.38}$, we conclude by Theorem 1 that all solutions, except those belonging to the stable manifold, converge towards the ω-limit set of the uncoupled system which is a globally attractive limit cycle for $k \gtrsim k^*$.

Simulation results for a network of 3 oscillators coupled according to (22) are presented in Figure 4. In this simulation, we used the same parameter values as for the S_N-symmetry case except for the coupling strength K which was chosen equal to 6 since the synchronization threshold is $\frac{k-k^*_{passive}}{0.38} = \frac{3-1}{0.38} = 5.23$. The initial conditions are chosen at random. The corresponding superposition of the oscillators state-spaces is represented on Figure 5.

6. CONCLUSIONS

In this paper, we show how incremental dissipativity may be used to extend the global stability analysis of a limit cycle existing for an isolated system to situations when such identical systems are arranged in particular networks.

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