Imperial College London

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Department of Bioengineering

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Training coursework 3

Implementation of the stochastic Euler algorithm

We will now implement an Euler algorithm for computing the numerical solutions of the *Stochastic Differential Equation*

$$dx = -kxdt + \sigma dW, \tag{1}$$

where σ is the amplitude of the random noise process dW.

Take $k = 3/16,\ h = 0.01,\ x(0) = 6,\ t \in [0, 10]$ and $\sigma = 0.2$ and write Matlab code to solve:

$$x(t+h) = x(t) + h\,[-k\,x(t)] + \sigma\sqrt{h} * \texttt{randn},$$

where `randn` is a Matlab function that generates a random number drawn from a *Gaussian distribution* of mean zero and unit variance (see `help randn`).

1. (a) Run your code 20 times and superimpose the plots of $x(t)$ as a function of time.
   (b) Using these 20 trajectories, calculate the average trajectory as a function of time and use the mean squared error to compare it to the analytical solution obtained for the deterministic system (i.e. where $\sigma = 0$).
2. (a) Run your code with $x(0) = 0$, $\sigma = 0.1$ and $h = 0.01$ for a long time, $T \gg 10$. (Note the initial condition is zero now.) Plot the *histogram* of the values of $x(T)$.
   (b) Repeat this calculation but now with $\sigma = 5$ and plot the corresponding *histogram*.
   (c) Use Matlab to calculate the mean and standard deviation of the two distributions obtained above. Explain the difference in the width of the histograms for the two values of σ.

> In this coursework you may need to use the following Matlab commands: `randn`, `function`, `plot`, `mean`, `std`, `hist`. You can check the Matlab help by using `help COMMAND`.